Joint Turbo Equalization and Multiuser Detection of MC-CDMA Signals with Low Envelope Fluctuations

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***Abstract*—In this paper we consider the uplink transmission in MC-CDMA (MultiCarrier - Coded Division Multiple Access) systems. As other multicarrier signals, MC-CDMA signals have high envelope fluctuations and a high PMEPR (Peak-to-Mean Envelope Power Ratio) which leads to amplification difficulties. This is particularly important for the uplink transmission, since an efficient, low-cost power amplification is desirable at the MTs (Mobile Terminals). Moreover, the transmission over time-dispersive channels destroys the orthogonality between spreading codes, which might lead to significant MAI (Multiple Access Interference) levels.**

To reduce the envelope fluctuations of the transmitted signals, while maintaining the spectral efficiency, the MC-CDMA signal associated to each MT (Mobile Terminal) is submitted to a clipping device, followed by a frequency-domain filtering operation. However, the nonlinear distortion effects can be high when an MC-CDMA transmitter with reduced envelope fluctuations is intended (e.g., a small clipping level and/or when successive clipping and filtering operations are employed).

In this paper, we define an iterative receiver that jointly performs a turbo-MUD (MultiUser Detection) and the estimation and cancelation of the nonlinear distortion effects.

Our performance results show that the proposed receiver structure allows good performances, very close to the linear receiver ones, even for high system load and/or when a PMEPR as low as 1.7 dB is intended for each MT.

***Index Terms*—Multicarrier-Coded Division Multiple Access (MC-CDMA), turbo equalization, multiuser detection, nonlinear distortion.**

I. INTRODUCTION

MC-CDMA schemes (MultiCarrier - Coded Division Multiple Access) combine an OFDM modulation (Orthogonal Frequency Division Multiplexing) [1] with a CDMA scheme [2]. Spreading is performed in the frequency-domain and MC-CDMA schemes are promising candidates for future broadband wireless systems. Since the transmission over time-dispersive channels destroys the orthogonality between spreading codes, an FDE (Frequency-Domain Equalizer) optimized under an MMSE criterion (Minimum Mean-Squared Error) is usually employed at the receiver [3], [4]. Since an MMSE FDE does not perform an ideal channel inversion, we are not able to fully orthogonalize the different spreading codes, which can lead to severe interference levels, especially for fully loaded systems and/or when different powers are assigned to different spreading codes. To improve the performance several turbo-MUD receivers (Multiuser Detection) were proposed [5]-[7]. In [5], the use of soft information for interference cancelation is exploited in MC-CDMA systems. An iterative semiblind receiver for coded MC-CDMA systems, able to deal with intra-cell and intercell interference, is proposed in [6]. A novel low-complexity PIC (Parallel Interference Cancelation) receiver for turbo coded MC-CDMA systems is also investigated in [7]. A promising iterative receiver for multicode MC-CDMA signals was proposed in [8], based on the IB-DFE (Iterative block Decision Feedback Equalizer) concept [9]-[11], allowing significant performance improvements especially for fully loaded systems and high spreading factors.

As with other multicarrier schemes, MC-CDMA signals have strong envelope fluctuations and high PMEPR values (Peak-to-Mean Envelope Power Ratio), which lead to amplification difficulties. For this reason, it is desirable to reduce the envelope fluctuations of the transmitted signals. This is particularly important for the uplink transmission, since an efficient, low-cost power amplification is desirable at the MT (Mobile Terminal). Several techniques have been recommended for reducing the envelope fluctuations of multicarrier signals [12]-[15]. A promising approach is to employ clipping techniques, combined with a frequency-domain filtering so as to reduce the envelope fluctuations of the transmitted signals while maintaining the spectral occupation of conventional schemes [15]. By repeating the clipping and filtering procedure we can further reduce the PMEPR of the transmitted signals. However, the nonlinear distortion effects can be severe when a transmission with very low PMEPR values is intended [15], [16]. By using iterative receivers with estimation and cancelation of nonlinear distortion effects we can improve significantly the performance in the presence of strong nonlinear distortion effects [17]-[19]. However, for low SNR (Signal-to-Noise Ratio) the error decisions might lead to error propagation effects, since errors in the estimation of nonlinear distortion effects can preclude its cancelation. This is particularly serious for high system load and/or when we decrease the clipping level to reduce further the PMEPR of the transmitted signals [19]. In [16], an enhanced receiver structure for the downlink transmission of MC-CDMA has considered, where an iterative estimation and cancelation of nonlinear distortion effects is

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This work was supported in part by Fundação para a Ciência e Tecnologia (pluriannual funding, U-BOAT project PTDC/EEA-TEL/67066/2006 and the FCT/POCI 2010 research grant SFRH/BD/24520/2005).

carried out.

In this paper we consider the uplink transmission in MC-CDMA systems. We modify the approach of [19] so as to cope with it's major limitations, namely the poor performance in the presence of severely nonlinear distortion effects and/or at low SNR, the envelope fluctuations regrowth after the filtering operation and the limitations of using the soft decisions values to obtain an estimate of the nonlinear distortion effects. To allow an efficient power amplification, the PMEPR-reducing techniques of [15] are adopted by each MT, which can be repeated several times. The BS (Base Station) has several receive antennas, so as to reduce the transmit power requirements of each MT. To avoid error propagation effects in the typical region of operation we use channel decoder outputs in the feedback loop, in a turbo-like fashion (a similar approach was proposed for OFDM schemes [20]). We define an iterative receiver that jointly performs a turbo-MUD and the estimation and cancelation of nonlinear distortion effects, for each iteration, that are inherent to the transmitted signals. To improve the performance at low SNR we consider a threshold-based cancelation.

This paper is organized as follows: the linear transmitter and receiver structures are described in Sec. II. In Sec. III we describe the nonlinear transmitter and receiver structures proposed in this paper. Implementation complexity issues are discussed in Sec. IV. Sec. V presents a set of performance results and Sec. VI is concerned with the conclusions of the paper.

Throughout this paper we will employ the following notation: bold letters $\mathbf{A}$ denote matrixes or vectors; $\mathbf{I}_N$ denote the N-by-N identity matrix; $\mathbf{0}_{N\times M}$ denote the N-by-M zero matrix; $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$ and diag$(\cdot)$ denote the transpose, conjugate, hermitian and diagonal matrix, respectively; $[\mathbf{A}]_{n,m}$ denote the element of line n and column m of matrix $\mathbf{A}$. x mod y is the reminder of division of x by y and $\delta_{k,k'}=1$ if $k=k'$ and 0 otherwise.

II. Linear Transmitter and Receiver Structures

In this paper we consider the uplink transmission in MC-CDMA systems employing frequency-domain spreading. The frequency-domain block to be transmitted by the pth MT is $\{S_{k,p}; k=0,1,\ldots,N-1\}$, where $N=KM$, with K denoting the spreading factor and M the number of data symbols for that MT. The frequency-domain symbols are given by $S_{k,p}=\xi_p C_{k,p} A_{k \bmod M,p}$, where ξ_p is an appropriate weighting coefficient that accounts for the propagation losses and $\{A_{k,p}; k=0,1,\ldots,M-1\}$ is the block of data symbols associated to the pth MT, assumed to be selected from a given constellation (in fact, different constellations could be selected for different data symbols, as when loading techniques are employed; in that case, the power associated to different data symbols is not necessarily the same). $\{C_{k,p}; k=0,1,\ldots,N-1\}$ is the corresponding spreading sequence[1] (a pseudo-random spreading is assumed, with $C_{k,p}$ belonging to a QPSK constellation; without loss of generality, it is assumed that $|C_{k,p}|=1$).

[1]This corresponds to uniformly spread the chips associated to a given symbol within the transmission band, i.e., to employ a rectangular interleaver with dimensions $K\times M$.

As usual, it is assumed that the length of the CP (Cyclic Prefix) is higher than the length of the overall channel impulse response. We will assume that the BS has L receive antennas and the received time-domain block associated to the lth diversity branch, after discarding the samples associated to the CP, is $\{y_n^{(l)}; n=0,1,\ldots,N-1\}$. The corresponding frequency-domain block $\{Y_k^{(l)}; k=0,1,\ldots,N-1\}$ (i.e., the length-N DFT (Discrete Fourier Transform) of the block $\{y_n^{(l)}; n=0,1,\ldots,N-1\}$) is

$$\begin{aligned} Y_k^{(l)} &= \sum_{p=1}^{P} S_{k,p}\, H_{k,p}^{Ch(l)} + N_k^{(l)} \\ &= \sum_{p=1}^{P} A_{k \bmod M,p}\, C_{k,p}\, \xi_p\, H_{k,p}^{Ch(l)} + N_k^{(l)} \\ &= \sum_{p=1}^{P} A_{k \bmod M,p}\, H_{k,p}^{(l)} + N_k^{(l)} \end{aligned} \tag{1}$$

with $H_{k,p}^{Ch(l)}$ denoting the channel frequency response between the pth MT and the lth diversity branch, at the kth subcarrier, $N_k^{(l)}$ the corresponding channel noise and $H_{k,p}^{(l)}=\xi_p H_{k,p}^{Ch(l)} C_{k,p}$. To detect the kth symbol of the pth MT we will use the set of subcarriers $\Psi_k=\{k, k+M, \ldots, k+(K-1)M\}$.

By defining $\mathbf{Y}(k)=[\mathbf{Y}^{(1)}(k)\cdots\mathbf{Y}^{(L)}(k)]^T$, with $\mathbf{Y}^{(l)}(k)=[Y_k^{(l)}\ \cdots\ Y_{k+(K-1)M}^{(l)}]$ denoting the line vector with the received samples associated to the set of frequencies Ψ_k, for the lth antenna, and $\mathbf{A}(k)=[A_{k \bmod M,1}\ldots A_{k \bmod M,P}]^T$, we have

$$\mathbf{Y}(k)=\mathbf{H}^T(k)\mathbf{A}(k)+\mathbf{N}(k) \tag{2}$$

where $\mathbf{N}(k)=[\mathbf{N}^{(1)}(k)\cdots\mathbf{N}^{(L)}(k)]^T$, with $\mathbf{N}^{(l)}(k)=[N_k^{(l)}\ \cdots\ N_{k+(K-1)M}^{(l)}]$ denoting the line vector with the noise samples associated to the set of frequencies Ψ_k, for the lth antenna. In (2), $\mathbf{H}(k)$ is the size-$P\times KL$ overall channel frequency response matrix associated to $\mathbf{A}(k)$, i.e.,

$$\mathbf{H}(k)=\left[\mathbf{H}^{(1)}(k)\ \cdots\ \mathbf{H}^{(L)}(k)\right]=[\mathbf{H}_1(k)\ \cdots\ \mathbf{H}_P(k)]^T \tag{3}$$

where

$$\mathbf{H}^{(l)}(k)=\begin{bmatrix} H_{k,1}^{(l)} & \cdots & H_{k+(K-1)M,1}^{(l)} \\ \vdots & & \vdots \\ H_{k,P}^{(l)} & \cdots & H_{k+(K-1)M,P}^{(l)} \end{bmatrix} \tag{4}$$

is a size-$P\times K$ matrix with lines associated to the different MTs and columns associated to the set of frequencies Ψ_k, for the lth antenna, and

$$\begin{aligned} \mathbf{H}_p(k) &= \Big[H_{k,p}^{(1)}\ \cdots\ H_{k+(K-1)M,p}^{(1)} \\ &\qquad \cdots\ H_{k,p}^{(L)}\ \cdots\ H_{k+(K-1)M,p}^{(L)}\Big]^T \end{aligned} \tag{5}$$

is a column vector associated to the pth MT.

This receiver can be regarded as an iterative multiuser receiver (IMUD) with PIC, as depicted in Fig. 1a). The receiver can be described as follows. For a given iteration,

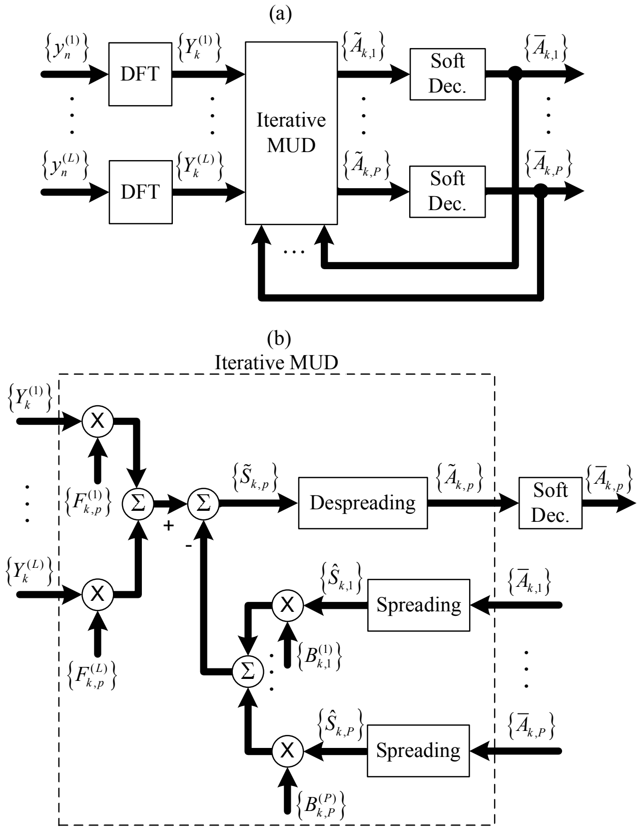

Fig. 1. Iterative receiver for a linear transmitter (a) and detail of the detection of the pth MT (b).

the detection of $\mathbf{A}(k)$ employs the structure depicted in Fig. 1b), where we have L feedforward filters (one for each receive antennas) and P feedback loops. The feedforward filters are designed to minimize the MAI (Multiple Access Interference) that cannot be canceled by the feedback loops. For the first iteration we do not have any information about the MT's symbols and the receiver reduces to a linear multiuser receiver.

For each iteration, the samples vector $\tilde{\mathbf{A}}(k)$ associated to $\mathbf{A}(k)$, is given by

$$\tilde{\mathbf{A}}(k) = \mathbf{F}^T(k)\mathbf{Y}(k) - \mathbf{B}^T(k)\overline{\mathbf{A}}(k) \tag{6}$$

where $\tilde{\mathbf{A}}(k)$ is defined as $\mathbf{A}(k)$, $\mathbf{F}(k)$ is a size-$KL \times P$ matrix with the feedforward coefficients given by

$$\mathbf{F}(k) = \left[\mathbf{F}^{(1)}(k) \ \cdots \ \mathbf{F}^{(L)}(k)\right]^T = [\mathbf{F}_1(k) \ \cdots \ \mathbf{F}_P(k)] \tag{7}$$

where

$$\mathbf{F}^{(l)}(k) = \begin{bmatrix} F^{(l)}_{k,1} & \cdots & F^{(l)}_{k,P} \\ \vdots & & \vdots \\ F^{(l)}_{k+(K-1)M,1} & \cdots & F^{(l)}_{k+(K-1)M,P} \end{bmatrix} \tag{8}$$

and

$$\begin{aligned} \mathbf{F}_p(k) \ = \ & \Big[F^{(1)}_{k,p} \ \cdots \ F^{(1)}_{k+(K-1)M,p} \\ & \cdots \ F^{(L)}_{k,p} \ \cdots \ F^{(L)}_{k+(K-1)M,p}\Big]^T, \end{aligned} \tag{9}$$

and $\mathbf{B}(k)$ is a size-$P \times P$ matrix with the feedback coefficients given by

$$\mathbf{B}(k) = [\mathbf{B}_1(k) \ \cdots \ \mathbf{B}_P(k)] = \left[\mathbf{B}^{(1)}(k) \ \cdots \ \mathbf{B}^{(P)}(k)\right]^T \tag{10}$$

with $\mathbf{B}_p(k) = [\ B^{(1)}_{k,p} \ \cdots \ B^{(P)}_{k,p}\ ]^T$ and $\mathbf{B}^{(p)}(k) = [\ B^{(p)}_{k,1} \ \cdots \ B^{(p)}_{k,P}\ ]$. $\overline{\mathbf{A}}(k)$ is the vector with the "soft decisions" of $\mathbf{A}(k)$ from the multiuser detector, obtained at the previous iteration, i.e., their components $\overline{A}_{k,p}$ are the expected value of $A_{k,p}$ conditioned to the multiuser detector output, at each iteration.

In [19] it is shown that, for a QPSK constellation under a Gray mapping rule, $\overline{A}_{k,p}$ is given by

$$\overline{A}_{k,p} = \tanh\left(\frac{L^I_{k,p}}{2}\right) + j\tanh\left(\frac{L^Q_{k,p}}{2}\right), \tag{11}$$

where

$$L^I_{k,p} = \frac{2}{\sigma_p^2}\tilde{A}^I_{k,p} \tag{12}$$

and

$$L^Q_{k,p} = \frac{2}{\sigma_p^2}\tilde{A}^Q_{k,p} \tag{13}$$

are the LLRs of the "in-phase bit" and the "quadrature bit", associated to $A^I_{k,p} = \mathrm{Re}\{A_{k,p}\}$ and $A^Q_{k,p} = \mathrm{Im}\{A_{k,p}\}$, respectively, with

$$\sigma_p^2 = \frac{1}{2}E\left[|A_{k,p} - \tilde{A}_{k,p}|^2\right] \approx \frac{1}{2M}\sum_{k=0}^{M-1} E\left[|\hat{A}_{k,p} - \tilde{A}_{k,p}|^2\right], \tag{14}$$

and $\hat{A}_{k,p}$ denoting the "hard decisions" associated to $\tilde{A}_{k,p}$. It should be pointed out that larger PAM and QAM constellations can be expressed as a combination of the corresponding bits [21]. Therefore, if the different bits are uncorrelated (the usual case for uncoded scenarios, as well as for coded scenarios with appropriate interleavers), the average symbol values for PAM or QAM constellations can be easily obtained from the average values associated to the corresponding bits [22].

The hard decisions $\hat{A}^I_{k,p} = \pm 1$ and $\hat{A}^Q_{k,p} = \pm 1$ are defined according to the signs of $L^I_{k,p}$ and $L^Q_{k,p}$, respectively;

$$\rho^I_{k,p} = \tanh\left(\frac{|L^I_{k,p}|}{2}\right) \tag{15}$$

and

$$\rho^Q_{k,p} = \tanh\left(\frac{|L^Q_{k,p}|}{2}\right) \tag{16}$$

can be regarded as the reliabilities associated to the "in-phase" and "quadrature" bits of the kth symbol of the pth MT (naturally, $0 \leq \rho^I_{k,p} \leq 1$ and $0 \leq \rho^Q_{k,p} \leq 1$). For the first iteration, $\rho^I_{k,p} = \rho^Q_{k,p} = 0$ and $\overline{A}_{k,p} = 0$; after some iterations and/or when the SNR is high, typically $\rho^I_{k,p} \approx 1$ and $\rho^Q_{k,p} \approx 1$, leading to $\overline{A}_{k,p} \approx \hat{A}_{k,p}$. We can also define the blockwise reliability

$$\rho_p = \frac{1}{M}\sum_{k=0}^{M-1}\frac{E[A^*_{k,p}\hat{A}_{k,p}]}{E[|A_{k,p}|^2]} = \frac{1}{2M}\sum_{k=0}^{M-1}\left(\rho^I_{k,p} + \rho^Q_{k,p}\right). \tag{17}$$

To avoid error propagation effects, we can also define a receiver (Turbo-MUD) that, as turbo equalizers, employs the “soft decisions” from the SISO channel decoder outputs (Soft-In, Soft-Out) instead of the “soft decisions” from the multiuser detector. The SISO block, that can be implemented as defined in [23], provides the LLRs (LogLikelihood Ratios) of both the ”information bits” and the ”coded bits”. The input of the SISO block are LLRs of the ”coded bits” at the multiuser detector.

Derivation of the Receiver Coefficients

To simplify the computation of $\mathbf{F}(k)$ and $\mathbf{B}(k)$ it is assumed that [9], [10]

$$\hat{A}_{k,p} \approx \rho_p A_{k,p} + \Delta_{k,p} \tag{18}$$

where $\Delta_{k,p}$ denotes the error associated to the kth symbol of the pth MT, with $E[\Delta_{k,p}] \approx 0$, $E[\Delta_{k,p} A_{k',p}] \approx 0$, regardless of k and k', and $E[|\Delta_{k,p}|^2] = (1-\rho_p^2)E[|A_{k,p}|^2]$. We will also assume [24] that

$$\overline{A}_{k,p} \approx \rho_p \hat{A}_{k,p} \approx \rho_p^2 A_{k,p} + \rho_p \Delta_{k,p}. \tag{19}$$

Although (18) and (19) may be considered rude approximations, they allow simple computation of $\hat{A}_{k,p}$ and $\overline{A}_{k,p}$ (naturally, for $\rho_p = 1$ and $\rho_p = 0$ (18) and (19) are exact). In matrix notation, (19) takes the form[2]

$$\overline{\mathbf{A}}(k) = \mathbf{P}^2 \mathbf{A}(k) + \mathbf{P}\boldsymbol{\Delta}(k), \tag{20}$$

with $\boldsymbol{\Delta}(k) = [\Delta_{k,1} \cdots \Delta_{k,P}]^T$ and $\mathbf{P} = \mathrm{diag}(\rho_1, \cdots, \rho_p)$.

By combining (2), (6) and (20), we obtain, after some straightforward manipulation,

$$\begin{aligned}\tilde{\mathbf{A}}(k) \;=\; & \underbrace{\boldsymbol{\Gamma}(k)\mathbf{A}(k)}_{\text{Useful signal}} \\ & + \underbrace{\left(\mathbf{F}^T(k)\ \mathbf{H}^T(k) - \mathbf{B}^T(k)\ \mathbf{P}^2 - \boldsymbol{\Gamma}(k)\right)\mathbf{A}(k)}_{\text{Residual Interference}} \\ & - \underbrace{\mathbf{B}^T(k)\mathbf{P}\boldsymbol{\Delta}(k)}_{\text{"Noise" due to feedback errors}} + \underbrace{\mathbf{F}^T(k)\ \mathbf{N}(k)}_{\text{Channel noise}},\end{aligned} \tag{21}$$

where $\boldsymbol{\Gamma}(k) = [\boldsymbol{\Gamma}_1(k) \cdots \boldsymbol{\Gamma}_P(k)] = \mathrm{diag}(\gamma_1, \cdots, \gamma_P)$ with

$$\gamma_p = \frac{1}{M}\mathbf{F}_p^T(k)\mathbf{H}_p(k). \tag{22}$$

γ_p can be regarded as the average overall channel frequency response, for the pth MT, after the feedforward filter $\mathbf{F}_p(k)$. If we have reliable estimates of the transmitted block, the feedback filter can then be used to remove this residual interference.

The feedforward and feedback matrixes, $\mathbf{F}(k)$ and $\mathbf{B}(k)$, respectively, are chosen so as to maximize the SINR (Signal-to-Interference plus Noise Ratio) for all MTs, at a particular iteration. For the pth MT the SINR is defined as

$$SINR_p = \frac{E[|\gamma_p A_{k,p}|^2]}{E[|\Theta_{k,p}|^2]}, \tag{23}$$

where $\Theta_{k,p} = \tilde{A}_{k,p} - A_{k,p}$ denotes the overall error for the kth symbol of the pth MT, that includes both the channel noise and the residual interference. By defining the overall error column vector associated to the symbols of all MTs, $\boldsymbol{\Theta}(k) = \tilde{\mathbf{A}}(k) - \mathbf{A}(k)$, the maximization of $\{SINR_p, p = 1, ..., P\}$ is equivalent to the minimization of

$$\begin{aligned}E\left[|\boldsymbol{\Theta}(k)|^2\right] = & E\left[\left|\left(\mathbf{F}^T(k)\mathbf{H}^T(k) - \mathbf{B}^T(k)\mathbf{P}^2 - \mathbf{I}_P\right)\mathbf{A}(k)\right|^2\right] \\ & + E\left[\left|\mathbf{B}^T(k)\mathbf{P}\boldsymbol{\Delta}(k)\right|^2\right] + E\left[\left|\mathbf{F}^T(k)\mathbf{N}(k)\right|^2\right]\end{aligned} \tag{24}$$

conditioned to $\gamma_p = 1$.

This minimization can be performed by employing the Lagrangian's multipliers method. For this purpose, we can define the matrix of Lagrangian functions

$$\mathbf{J} = E\left[|\boldsymbol{\Theta}(k)|^2\right] + \left(\boldsymbol{\Gamma}(k) - \mathbf{I}_P\right)\boldsymbol{\Lambda}, \tag{25}$$

where $\boldsymbol{\Lambda} = [\boldsymbol{\Lambda}_1 \cdots \boldsymbol{\Lambda}_P] = \mathrm{diag}(\lambda_1, \lambda_2, ..., \lambda_P)$ is the Lagrange's multipliers matrix, and assume that the optimization is carried out under $\boldsymbol{\Gamma}(k) = \mathbf{I}_P$. It can be shown that the optimum feedback and feedforward matrices are given by

$$\mathbf{B}(k) = \mathbf{H}(k)\mathbf{F}(k) - \mathbf{I}_P \tag{26}$$

and

$$\mathbf{F}(k) = \left[\mathbf{H}^H(k)\left(\mathbf{I}_P - \mathbf{P}^2\right)\mathbf{H}(k) + \frac{\sigma_N^2}{\sigma_A^2}\mathbf{I}_{KL}\right]^{-1}\mathbf{H}^H(k)\mathbf{Q}, \tag{27}$$

respectively, where σ_N^2 denotes the variance of the noise terms, σ_A^2 the variance of the data symbols and the normalization matrix

$$\mathbf{Q} = \mathrm{diag}(Q_1, ..., Q_P) = \mathbf{I}_P - \mathbf{P}^2 - \frac{1}{2M\sigma_A^2}\boldsymbol{\Lambda} \tag{28}$$

ensures that $\boldsymbol{\Gamma}(k) = \mathbf{I}_P$.

For the first iteration, we do not have data estimates for the different users, so $\mathbf{P} = 0$ and the feedback coefficients are zero. In this case, (27) reduces to a linear MUD receiver.

In the Appendix, it is shown that the optimum feedforward matrix can also be written as

$$\mathbf{F}(k) = \mathbf{H}^H(k)\mathbf{V}(k)\mathbf{Q}, \tag{29}$$

with $\mathbf{V}(k)$ given by

$$\mathbf{V}(k) = \left[(\mathbf{I}_P - \mathbf{P}^2)\mathbf{H}(k)\mathbf{H}^H(k) + \frac{\sigma_N^2}{\sigma_A^2}\mathbf{I}_N(k)\right]^{-1}. \tag{30}$$

The computation of the feedforward coefficients from (29)-(30) is simpler than the direct computation from (27), especially when $P < KL$.

III. Nonlinear Transmitter and Receiver Structures

To reduce the envelope fluctuations of the transmitted signals, we employ the transmitter structure depicted in Fig. 2a), which is based on the nonlinear signal processing schemes proposed in [15] for reducing the PMEPR of OFDM signals while maintaining the spectral efficiency of conventional OFDM schemes. Each time-domain sample is submitted to a nonlinear device so as to reduce the envelope fluctuations on the transmitted signal (see Fig. 2b)). We assume that the

[2] $\mathbf{P}$ (capital ρ) should not be confused with P (number of users).

$$\mathbf{H}^{Ch}(k) = \begin{bmatrix} \begin{bmatrix} \mathbf{H}_k^{Ch(1)} & \mathbf{0}_{1\times P} & \cdots & \mathbf{0}_{1\times P} \\ \mathbf{0}_{1\times P} & \mathbf{H}_{k+M}^{Ch(1)} & \cdots & \mathbf{0}_{1\times P} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_{1\times P} & \mathbf{0}_{1\times P} & \cdots & \mathbf{H}_{k+(K-1)M}^{Ch(1)} \end{bmatrix} & \cdots & \mathbf{0}_{K\times KP} \\ \vdots & \ddots & \vdots \\ \mathbf{0}_{K\times KP} & \cdots & \begin{bmatrix} \mathbf{H}_k^{Ch(L)} & \mathbf{0}_{1\times P} & \cdots & \mathbf{0}_{1\times P} \\ \mathbf{0}_{1\times P} & \mathbf{H}_{k+M}^{Ch(L)} & \cdots & \mathbf{0}_{1\times P} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_{1\times P} & \mathbf{0}_{1\times P} & \cdots & \mathbf{H}_{k+(K-1)M}^{Ch(L)} \end{bmatrix} \end{bmatrix}^T \tag{31}$$

nonlinear device is an ideal envelope clipping with clipping level s_M. After a DFT operation the clipped signal is then submitted to a frequency-domain filtering procedure, through the set of multiplying coefficients $\{G_k, k = 0, 1, \ldots, N'-1\}$, in order to reduce the out-of-band radiation levels inherent to the nonlinear operation. Since the frequency-domain filtering procedure produces some envelope fluctuations regrowth, limiting the achievable PMEPR, clipping and filtering operations can be repeated iteratively so as to reduce further the PMEPR of the transmitted signals.

It is shown in [16] that the frequency-domain block to be transmitted by the pth MT $\{S_{k,p}^{Tx} = S_{k,p}^{C} G_k; k = 0, 1, \ldots, N'-1\}$ can be decomposed into the sum of two uncorrelated components, i.e., $S_{k,p}^{Tx} = \alpha_{k,p} S_{k,p} G_k + D_{k,p} G_k$, where $\alpha_{k,p}$ is a scalar factor, as defined in [15], and $\{D_{k,p}; k = 0, 1, \ldots, N-1\}$ is the frequency-domain block of nonlinear self-interference components associated to the pth MT. Throughout this paper we assume that $G_k = 1$ for the N in-band subcarriers and 0 for the $N'-N$ out-of-band subcarriers. In this case, $S_{k,p}^{Tx} = \alpha_{k,p} S_{k,p} + D_{k,p}$ for the in-band subcarriers and 0 for the out-of-band subcarriers. It can also be shown that $D_{k,p}$ is approximately Gaussian-distributed, with zero mean; moreover, $E[D_{k,p} D_{k',p}^*] = 2\sigma_{D,p}^2(k)\delta_{k,k'}$. For a transmitter with a single clipping operation $\sigma_{D,p}^2(k)$ can be computed analytically as defined in [15]; if successive clipping and filtering operations are employed then $\sigma_{D,p}^2(k)$ has to be obtained by simulation.

The performance of OFDM schemes submitted to nonlinear devices can be significantly improved by employing a receiver with iterative cancelation of nonlinear distortion effects [17], [18]. This concept can be extended to MC-CDMA, leading to the receiver structure of Fig. 3a). The basic idea behind this receiver is to use the estimates of the nonlinear distortion $D_{k,p}$, $\hat{D}_{k,p}$, provided by the preceding iteration to remove the nonlinear distortion effects in the received samples.

We will define the size-KLP vector $\mathbf{D}(k)$ as the concatenation of $[\mathbf{D}_k \ \mathbf{D}_{k+M} \ \ldots \ \mathbf{D}_{k+(K-1)M}]^T$ L times, where $\mathbf{D}_k = [D_{k,1} \ \ldots \ D_{k,P}]$, and $\hat{\mathbf{D}}(k)$ its estimate (obtained from the previous iteration). We also define the size-$KLP \times KL$ channel matrix $\mathbf{H}^{Ch}(k)$ given by (31), with $\mathbf{H}_k^{Ch(l)} = [H_{k,1}^{Ch(l)} \ H_{k,2}^{Ch(l)} \ \ldots \ H_{k,P}^{Ch(l)}]$.

Therefore, the received frequency-domain block vector,

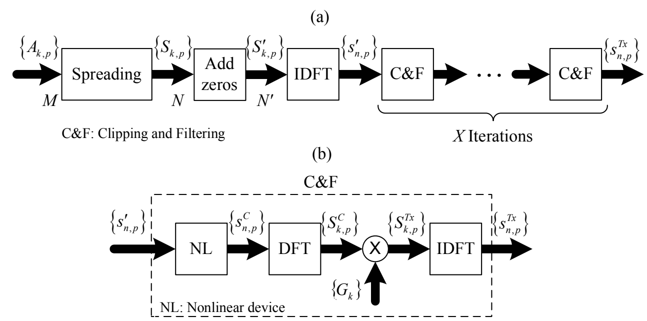

Fig. 2. Transmitter model considered in this paper (a) and detail of C&F block (b).

$\mathbf{Y}(k)$, which for nonlinear transmitters is

$$\mathbf{Y}(k) = \mathbf{H}^{Use^T}(k)\mathbf{A}(k) + \mathbf{H}^{Ch^T}(k)\mathbf{D}(k) + \mathbf{N}(k), \tag{32}$$

is replaced by the corrected block vector $\mathbf{Y}^{Corr}(k)$, given by

$$\begin{aligned} \mathbf{Y}^{Corr}(k) &= \mathbf{Y}(k) - \mathbf{H}^{Ch^T}(k)\hat{\mathbf{D}}(k) \\ &= \mathbf{H}^{Use^T}(k)\mathbf{A}(k) + \mathbf{H}^{Ch^T}(k)\mathbf{D}^{Res}(k) + \mathbf{N}(k), \end{aligned} \tag{33}$$

with

$$\mathbf{H}^{Use}(k) = \begin{bmatrix} \mathbf{H}^{Use(1)}(k) & \cdots & \mathbf{H}^{Use(L)}(k) \end{bmatrix}, \tag{34}$$

where the size-$P \times K$ matrix $\mathbf{H}^{Use(l)}(k)$, $l = 1, ..., L$, is given by

$$\mathbf{H}^{Use(l)}(k) = \begin{bmatrix} \alpha_{k,1} H_{k,1}^{(l)} & \cdots & \alpha_{k+(K-1)M,1} H_{k+(K-1)M,1}^{(l)} \\ \vdots & & \vdots \\ \alpha_{k,P} H_{k,P}^{(l)} & \cdots & \alpha_{k+(K-1)M,P} H_{k+(K-1)M,P}^{(l)} \end{bmatrix} \tag{35}$$

and $\mathbf{D}^{Res}(k) = \mathbf{D}(k) - \hat{\mathbf{D}}(k)$ is the residual nonlinear self-distortion vector.

For the derivation of the optimum feedforward and feedback matrixes, $\mathbf{F}(k)$ and $\mathbf{B}(k)$, respectively, we can use the same approach as we did for linear transmitters by employing the Lagrangian multipliers method. Thus, it follows that

$$\mathbf{B}(k) = \mathbf{H}^{Use}(k)\mathbf{F}(k) - \mathbf{I}_P \tag{36}$$

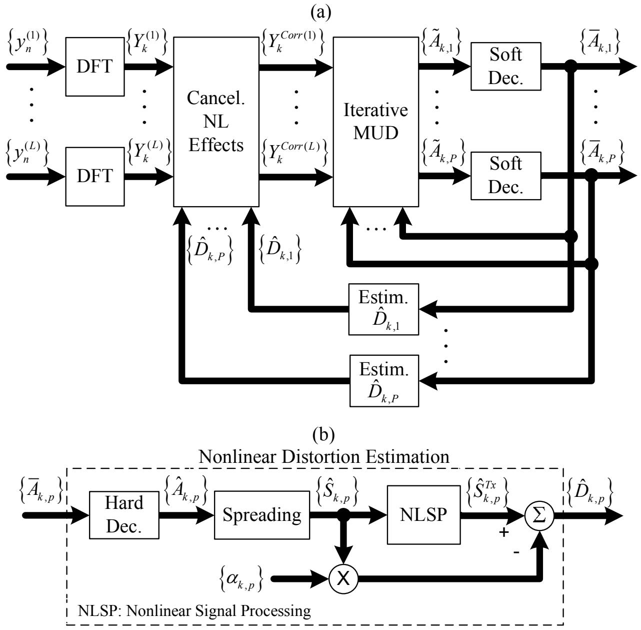

Fig. 3. Iterative receiver with cancelation of nonlinear distortion effects (a) and detail of the nonlinear distortion estimation (b).

and

$$\mathbf{F}(k) = \Big[\mathbf{H}^{Use^H}(k)\left(\mathbf{I}_P - \mathbf{P}^2\right)\mathbf{H}^{Use}(k) + \frac{1}{2\sigma_A^2}\mathbf{H}^{Ch^H}(k)\mathbf{R}_D(k)\mathbf{H}^{Ch}(k) + \frac{\sigma_N^2}{\sigma_A^2}\mathbf{I}_{KL}\Big]^{-1}\mathbf{H}^{Use^H}(k)\mathbf{Q}, \qquad (37)$$

where

$$\mathbf{R}_D(k) = E\left[\mathbf{D}^{Res^*}(k)\mathbf{D}^{Res^T}(k)\right]. \qquad (38)$$

For the first iteration the non-zero elements of $\mathbf{R}_D(k)$ are of the type $E[|D_{k,p}|^2] = 2\sigma_{D,p}^2(k)$. For the remaining iterations they have to be obtained by simulation. We assumed that

$$E[|D_{k,p}^{Res}|^2] \approx 2(1-\rho_p^x)E[|D_{k,p}|^2] \qquad (39)$$

where x is an exponent that depends on the adopted normalized clipping level s_M/σ, where $\sigma = E[|s'_{n,p}|^2]/2$, with $\{s'_{n,p}; n = 0, 1, ..., N'-1\}$ denoting the input samples.

For a given iteration, $\{\hat{D}_{k,p}; k = 0, 1, \ldots, N-1\}$ can be estimated from $\{\hat{A}_{k,p}; k = 0, 1, \ldots, M-1\}$ as follows (see Fig. 3b)): $\{\hat{A}_{k,p}; k = 0, 1, \ldots, M-1\}$ is re-spread to generate an estimate of the "block to be transmitted" $\{\hat{S}_{k,p}; k = 0, 1, \ldots, N-1\}$; $\{\hat{S}_{k,p}; k = 0, 1, \ldots, N-1\}$ is submitted to a replica of the nonlinear signal processing scheme employed in the pth transmitter so as to form the "transmitted block estimate" $\{\hat{S}_{k,p}^{Tx}; k = 0, 1, \ldots, N-1\}$; $\hat{D}_{k,p}$ is given by

$$\hat{D}_{k,p} = \hat{S}_{k,p}^{Tx} - \alpha_{k,p}\hat{S}_{k,p}. \qquad (40)$$

For small values of ρ_p the estimates are not reliable enough to allow accurate estimation of nonlinear distortion effects (in fact, $E[|D_{k,p}^{Res}|^2]$ can be larger than $E[|D_{k,p}|^2]$ for small values of ρ_p). For this reason, we only perform the estimation and cancelation of nonlinear distortion effects when $2(1-\rho_p^x) \leq 1$.

Naturally, for the first iteration, $\hat{\mathbf{D}}(k) = 0$ and $\mathbf{D}^{Res}(k) = \mathbf{D}(k)$. For the remaining iterations, $\mathbf{D}^{Res}(k) = \mathbf{D}(k) - \hat{\mathbf{D}}(k)$, where $\hat{\mathbf{D}}(k)$ is obtained from the previous iteration.

IV. Implementation Complexity Issues

The implementation complexity of our receivers can be measured in terms of number and size of DFT/IDFT operations, number of despreading/spreading operations, as well as the computation charge required for the calculation of the feedforward coefficients. In the case of the IMUD receiver we need L size-N DFT operations, one for each antenna, and a pair of despreading/spreading operations for the detection of each MT, at each iteration (except for the first iteration where only one despreading operation for each MT is required). If we have estimation and compensation of nonlinear effects, XP pairs of size-N DFT/IDFT operations (X is the number of C&F operations) for the detection of each MT, at each iteration are also needed. As for the computation of the feedforward coefficients, we need to invert the size-$P \times P$ matrix of (37) for each MT, at each iteration. Naturally, for slow-varying channels, this operations is not required for all blocks. In the case of Turbo-MUD receiver the SISO channel decoding needs to be implemented in the detection process of each MT, with the SOVA (Soft Output Viterbi Algorithm) instead of a conventional Viterbi algorithm. This can be the most complex part of Turbo-MUD receiver. Nevertheless, it should be pointed out that the implementation charge is concentrated in the BS, where increased power consumption and cost are not so critical.

It should also be pointed out that our receivers can be simplified with only negligible performance degradation by noting that whenever $\rho_p \approx 1$ for the pth MT at a given iteration, we already have reliable decisions for that MT and we can exclude it from the detection process in the next iteration.

When compared with a conventional MRC-PIC (Maximum Ratio Combining) receiver [25], our receivers are more complex, with an additional implementation complexity coming essentially from the computation of the feedforward coefficients and from the extra size-N DFT/IDFT operations required when estimation and compensation of nonlinear effects are employed. However, the main computational effort can be the one inherent to SISO channel decoding, something also required in MRC-PIC receivers with turbo decoding (moreover, MRC-PIC receivers have poor performance for high system load, while the Turbo-MUD receiver proposed in this paper have good performance even for fully loaded systems, as we will see in the next section).

V. Performance Results

In this section we present a set of performance results concerning the iterative receiver structures proposed in this paper for the uplink of MC-CDMA systems with frequency-domain spreading. We consider $M = 32$ data symbols for each user, corresponding to blocks with length $N = KM = 256$, plus an appropriate CP. QPSK constellations, with Gray mapping, are employed. To reduce the envelope fluctuations of the transmitted signals (and the PMEPR) while maintaining the spectral occupation of conventional MC-CDMA schemes, each MT employs the clipping techniques combined with a frequency-domain filtering proposed in [15] (the power amplifiers are assumed to be linear for the (reduced) dynamic range of the envelope fluctuations of the transmitted signals). The PMEPR of the transmitted signals (defined as in [15]) are shown in Table I, together with the corresponding average SIR values (Signal to nonlinear self-Interference Ratio). The receiver (i.e., the BS) knows the characteristics of the PMEPR-reducing signal processing technique employed by each MT.

We consider the power delay profile type C for the HIPERLAN/2 (HIgh PERformance Local Area Network) [26], with uncorrelated Rayleigh fading for the different MTs and for the different paths (similar results were obtained for other severely time-dispersive channels). The duration of the useful part of the block is 4μs and the CP has duration 1.25μs. We consider uncoded and coded BER performances under perfect synchronization and channel estimation conditions[3]. We consider the well-known rate-1/2, 64-state convolutional code with generators $1+D^2+D^3+D^5+D^6$ and $1+D+D^2+D^3+D^6$. The coded bits are interleaved before being mapped into QPSK symbols. The SISO decoder is implemented using the Max-Log-MAP approach. Unless otherwise stated, we consider $P = K = 8$ MTs, corresponding to a fully loaded scenario and $\xi_p = 1$ for all MTs, i.e., we have perfect "average power control" (in practice, there are some power fluctuations due to the fading). At the BS we have L uncorrelated receive antennas, for diversity purposes.

We will denote the receiver with soft decisions from the multiuser detector employed in the feedback loop as IMUD (Iterative MUD) and the receiver with soft decisions from the channel decoder outputs employed in the feedback loop as Turbo-MUD.

Let us first consider an uncoded case where we have nonlinear transmitters at each MT with a normalized clipping level, identical for all MTs, of $s_M/\sigma = 1.0$ and $s_M/\sigma = 0.5$. Figs. 4 and 5 show the average uncoded BER performance (i.e., the average over all MTs) for iterations 1 and 4 when $L = 1$ or 2, respectively (naturally, the first iteration corresponds to a linear receiver). For the sake of comparisons, we also include the performance for a linear transmitter and the SU (Single-User) performance, which, for the kth data symbol could be defined as

$$P_{b,SU,k} = E\left[Q\left(\sqrt{\frac{2E_b}{N_0}\frac{1}{K}\sum_{k'\in\Psi_k}\sum_{l=1}^{L}|H_{k'}^{(l)}|^2}\right)\right], \quad (41)$$

where the expectation is over the set of channel realizations (it is assumed that $E[|H_k^{(l)}|^2] = 1$ for any l). From Figs. 4 and 5 it is clear that the iterative receiver allows significant performance improvements relatively to the linear receiver, although, for $L = 1$, the performance is far from the SU performance, even after four iterations. Moreover, our simulation results showed that we can approach the SU performance, when we have diversity.

[3]It should be noted that perfect time synchronization between the blocks associated to different MTs is not required since some time mismatches can be absorbed by the CP.

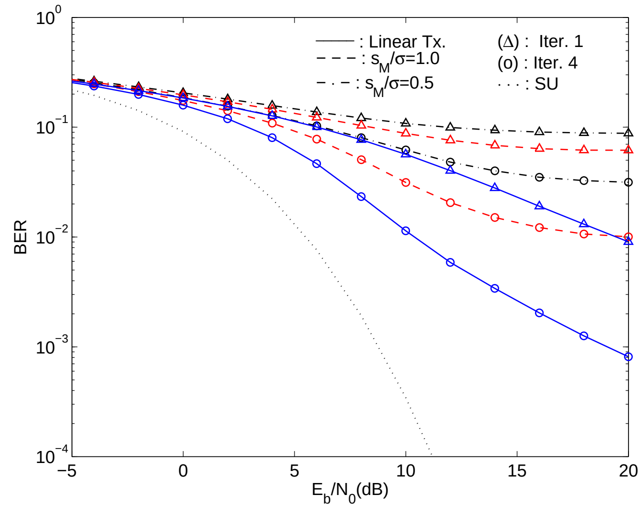

Fig. 4. Average uncoded BER performance for iterations 1 and 4 with $L = 1$ (better performances as we increase the number of iterations), as well as the corresponding SU performance, when linear and nonlinear transmitters with normalized clipping level of $s_M/\sigma = 1.0$ and 0.5 are considered.

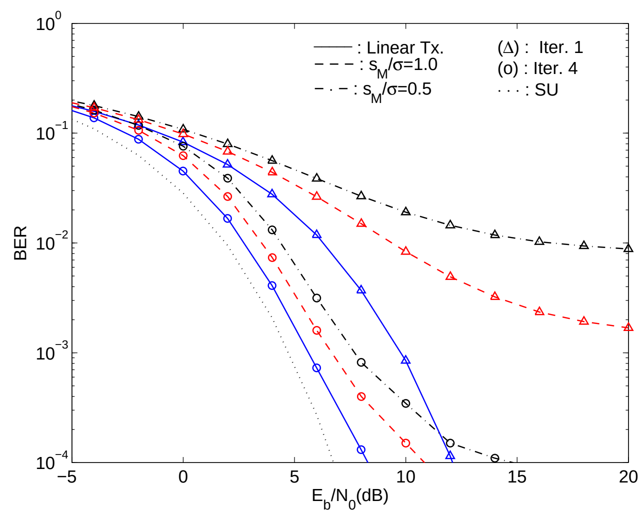

Fig. 5. As in Fig. 4 but with $L = 2$.

Lets consider now the impact of channel coding by assuming again that we have nonlinear transmitters at each MT with normalized clipping levels of $s_M/\sigma = 1.0$ and $s_M/\sigma = 0.5$. Figs. 6 and 7 show the average coded BER performance for iterations 1 and 4, again for $L = 1$ or 2, respectively, for either IMUD and Turbo-MUD receivers. As expected, the channel coding leads to significant performance improvements. Moreover, it is clear that the performance of the linear receiver is very poor, with high irreducible error floors due to the nonlinear distortion effects. This is especially serious for the case where $L = 1$. As we increase the number of iterations and/or we increase L improve significantly the performance, that can be close to the one obtained with linear transmitters if $L > 1$. We can also observe that the Turbo-MUD outperform the IMUD, especially when $L = 1$.

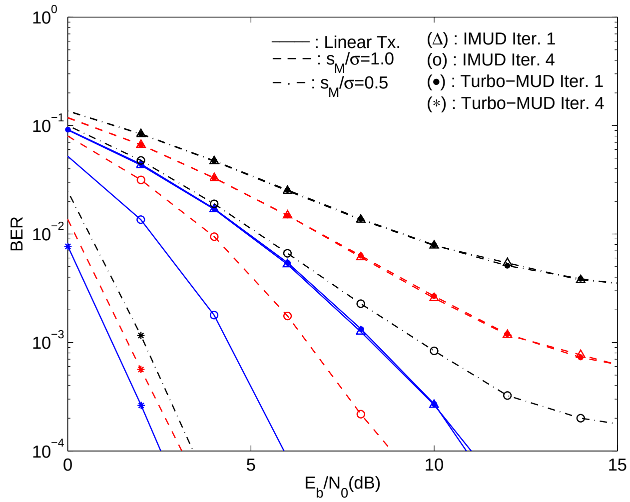

Fig. 6. Average coded BER performance for iterations 1 and 4 for either IMUD and Turbo-MUD receivers with $L = 1$, when linear and nonlinear transmitters with normalized clipping level of $s_M/\sigma = 1.0$ and 0.5 are considered.

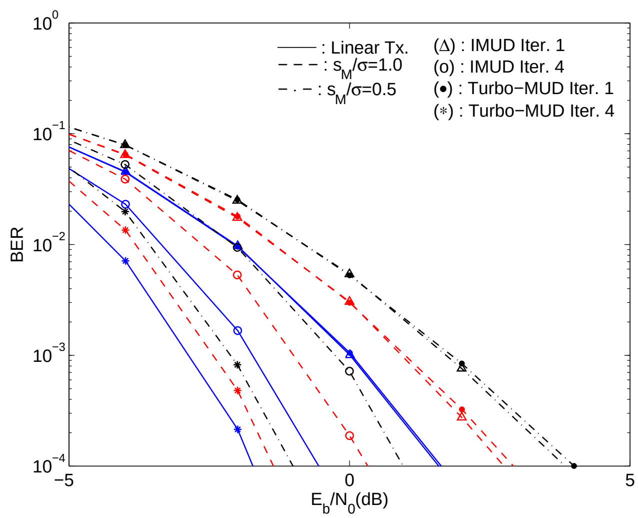

Fig. 7. As in Fig. 6 but with $L = 2$.

Lets consider a case where we want a transmission with a very low-PMEPR of the MC-CDMA signals, not only by assuming a very low clipping level at each MT, but also by repeating several times the clipping and filtering operations at each MT to further reduce the PMEPR of the transmitted signals while maintaining the spectral occupation of conventional MC-CDMA schemes (see Table I). Fig. 8 shows the average coded BER performance for iterations 1, 2 and 4 for Turbo-MUD receiver with $L = 1$, with 1, 2, 4 or 8 clipping and filtering iterations at the transmitter and a normalized clipping level of $s_M/\sigma = 0.5$. From this figure it is clear that the performance degradation associated to several clipping and filtering operations is very small when Turbo-MUD receivers with estimation and cancelation of nonlinear distortion effects are employed.

Finally, let us consider now a fully loaded scenario with $K = P = 4$, $L = 1$ and a normalized clipping level of

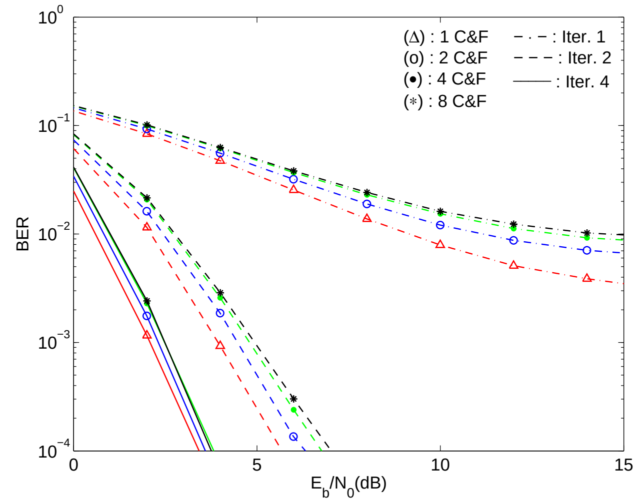

Fig. 8. Average coded BER performance for iterations 1, 2 and 4 for Turbo-MUD receiver with $L = 1$, with 1, 2, 4 or 8 clipping and filtering iterations at the transmitter and a normalized clipping level of $s_M/\sigma = 0.5$.

TABLE I
PMEPR AND SIR OF THE TRANSMITTED SIGNALS.

s_M/σ	**PMEPR (dB)**				**SIR (dB)**			
	Iterations				Iterations			
	1	2	4	8	1	2	4	8
0.5	4.1	3.0	2.0	1.7	8.8	7.9	7.5	7.3
1.0	4.4	3.4	2.4	2.1	11.1	10.0	9.3	9.1
1.5	5.0	4.0	3.2	2.9	14.6	13.0	12.2	12.0
2.0	5.7	4.9	4.2	4.0	19.4	17.4	16.3	15.9

$s_M/\sigma = 1.0$ where the signals associated to different users have different average power at the receiver. We will consider two classes of users, denoted by C_L and C_H, with two users in each class, where the average power of C_H users is 6dB above the average power of C_L users. Clearly, the C_L users face strong interference conditions. The coded BER performance for each C_L and C_H user as a function of E_b/N_0 of C_H users is shown in Figs. 9 and 10, respectively, for either IMUD and Turbo-MUD receivers. Once again, the iterative receiver allows significant performance gains, with the Turbo-MUD receiver outperforming the IMUD, especially for C_L users.

VI. CONCLUSIONS

In this paper we considered the uplink transmission in MC-CDMA systems employing clipping techniques so as to reduce the envelope fluctuations of the transmitted signals. We proposed an iterative receiver structure that combine turbo-MUD and estimation and cancelation of the nonlinear distortion effects that are inherent to the transmitted signals.

Our performance results showed that the use of the channel decoder outputs instead of the coded MUD outputs, in the feedback loop, allow a significant performance improve at low and moderate SNR, even for severely time-dispersive channels and/or when a very low-PMEPR MC-CDMA transmission is intended.

Although our receivers require additional implementation complexity, especially for the Turbo-MUD receiver, it should

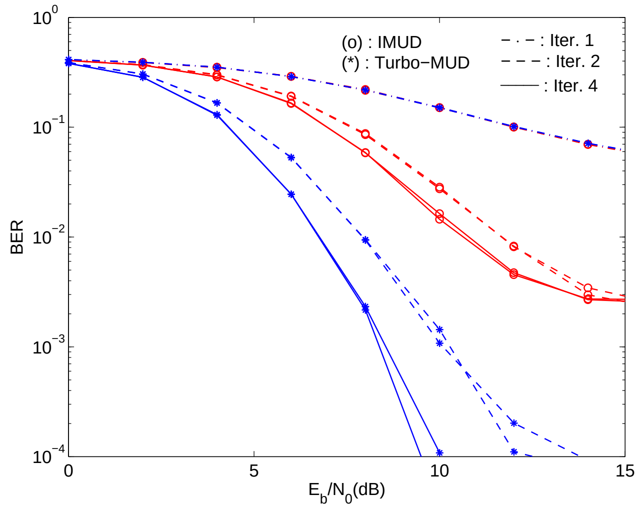

Fig. 9. Coded BER performance for each C_L user as a function of the E_b/N_0 of C_H users, when $P = K = 4$, $L = 1$ and $s_M/\sigma = 1.0$, for either IMUD or Turbo-MUD receivers (average power of C_L users 6dB below the average power of C_H users).

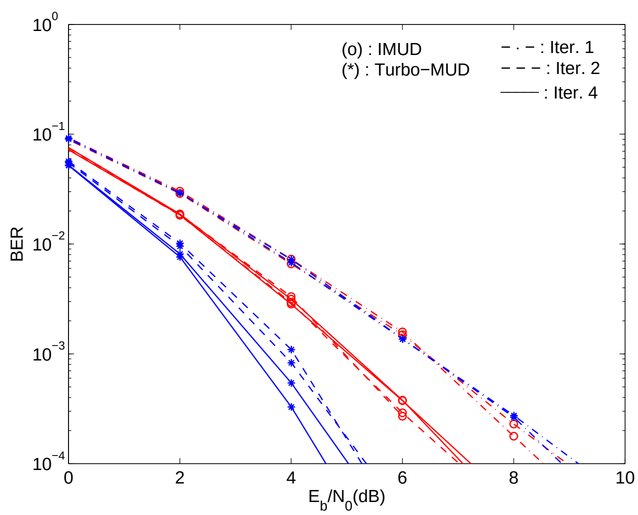

Fig. 10. As in Fig. 9 but for C_H users.

be pointed out that the implementation charge is concentrated in the BS, where increased power consumption and cost are not so critical. Having in mind the benefits of using efficient, low-cost power amplification at the MTs, the proposed techniques could mean an important implementation advantage for the MTs.

Appendix

In the following we will show that the optimum feedforward matrix $\mathbf{F}(k)$ given by (27) can be written as (29).

Lets rewrite (27) and (29) using non matricial notation:

$$\sum_{p'=1}^{P}(1-\rho_{p'}^2)H_{k,p'}^{(l)^*}\sum_{l'=1}^{L}H_{k,p'}^{(l')}F_{k,p}^{(l')}+\frac{\sigma_N^2}{\sigma_A^2}F_{k,p}^{(l)}=Q_pH_{k,p}^{(l)^*}, \quad (42)$$

$l=1,2,...,L$, where $Q_p=\gamma_p(1-\rho_p^2)-\lambda_p/(2\sigma_A^2M)$, and

$$F_{k,p}^{(l)}=\sum_{p'=1}^{P}H_{k,p'}^{(l)^*}[\mathbf{V}(k)]_{p,p'}[\mathbf{Q}]_{p,p'}, \quad (43)$$

respectively ($[\mathbf{A}]_{n,m}$ denote the element of line n and column m of matrix $\mathbf{A}$). Substituting (43) in (42) follows that

$$\begin{aligned}
&\sum_{p'=1}^{P}(1-\rho_{p'}^2)H_{k,p'}^{(l)^*}\sum_{l'=1}^{L}\sum_{p''=1}^{P}H_{k,p''}^{(l')^*}[\mathbf{V}(k)]_{p,p''}[\mathbf{Q}]_{p,p''}H_{k,p'}^{(l')}\\
&\quad+\frac{\sigma_N^2}{\sigma_A^2}\sum_{p''=1}^{P}H_{k,p''}^{(l)^*}[\mathbf{V}(k)]_{p,p''}[\mathbf{Q}]_{p,p''}=Q_pH_{k,p}^{(l)^*}\\
\Leftrightarrow&\sum_{p'=1}^{P}(1-\rho_{p'}^2)H_{k,p'}^{(l)^*}\sum_{l'=1}^{L}\sum_{p''=1}^{P}H_{k,p''}^{(l')^*}[\mathbf{V}(k)]_{p,p''}[\mathbf{Q}]_{p,p''}H_{k,p'}^{(l')}\\
&\quad+\frac{\sigma_N^2}{\sigma_A^2}\sum_{p'=1}^{P}\sum_{p''=1}^{P}H_{k,p''}^{(l)^*}[\mathbf{V}(k)]_{p,p''}[\mathbf{Q}]_{p,p''}\delta_{p',p''}=Q_pH_{k,p}^{(l)^*}\\
\Leftrightarrow&\sum_{p'=1}^{P}H_{k,p'}^{(l)^*}\Bigg[\sum_{p''=1}^{P}[\mathbf{V}(k)]_{p,p''}[\mathbf{Q}]_{p,p''}\\
&\quad\cdot\left((1-\rho_{p'}^2)\sum_{l'=1}^{L}H_{k,p''}^{(l')^*}H_{k,p'}^{(l')}+\frac{\sigma_N^2}{\sigma_A^2}\delta_{p',p''}\right)\Bigg]=Q_pH_{k,p}^{(l)^*},
\end{aligned} \quad (44)$$

$l=1,2,...,L$. From (30), which in non matricial notation is given by

$$\sum_{p''=1}^{P}[\mathbf{V}(k)]_{p,p''}\left((1-\rho_{p'}^2)\sum_{l'=1}^{L}H_{k,p''}^{(l')^*}H_{k,p'}^{(l')}+\frac{\sigma_N^2}{\sigma_A^2}\delta_{p',p''}\right)=\delta_{p,p'}, \quad (45)$$

we can easily see that the factor between brackets in (44) reduces to

$$\sum_{p''=1}^{P}[\mathbf{V}(k)]_{p,p''}[\mathbf{Q}]_{p,p''}\left((1-\rho_{p'}^2)\sum_{l'=1}^{L}H_{k,p''}^{(l')^*}H_{k,p'}^{(l')}+\frac{\sigma_N^2}{\sigma_A^2}\delta_{p',p''}\right)=[\mathbf{Q}]_{p,p'}\delta_{p,p'}. \quad (46)$$

Substituting the result above in (44) leads to

$$\sum_{p'=1}^{P}H_{k,p'}^{(l)^*}[\mathbf{Q}]_{p,p'}\delta_{p,p'}=Q_pH_{k,p}^{(l)^*}, \quad (47)$$

$l=1,...,L$, which completes the demonstration.

Acknowledgment

The authors would like to thank the reviewers for their helpful suggestions, based on which the quality of the paper has been improved.

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